A small cosmological constant from the modified Brans-Dicke theory − an interplay between different energy scales

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Abstract

In this paper we discuss a model in which the energy density, corresponding to the effective cosmological constant, after the $SU(2) \times U(1)$ symmetry breaking appears to be of the desired order of $10^{-48} \div 10^{-47} GeV^4$. The model contain two different energy scales, one of which is associated with the Higgs's vacuum expectation value. Another scale is of the order of $10^{21} GeV$ and defines the vacuum expectation value of the Brans-Dicke scalar field, non-minimally coupled to gravity, and sets the value of the Planck mass. Other (dimensionless) parameters are assumed not to contain hierarchical differences. The model is devoid of any fine-tuning and gives a small value of the effective cosmological constant even if the real "bare" cosmological constant is quite large.

1 Introduction

During the last years the problem of cosmological constant, its origin and small value attracts much attention. It is very likely that the vacuum energy is indeed a constant, and there are a lot of attempts to explain the existence of such constant vacuum energy – see, for example, reviews [1]–[10] and references therein. One of the most interesting questions is that about its extremely small value. Nevertheless, most of the mechanisms demand a fine tuning, and corresponding cancellations do not look quite natural.

In this paper we propose the model admitting any value of the real cosmological constant and providing a desired value of the effective cosmological constant. In the beginning, let us consider a simple action describing two interacting scalar fields with the action

$$ S = \int d^4x \sqrt{-g} \left[-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\partial^\mu h\partial_\mu h - \lambda_1 \left(\phi^2 - M^2\right)^2 - \lambda_2 \left(h^2 - \frac{v^2}{2}\right)^2 - \gamma\frac{h^8}{\phi^4} \right], \quad (1) $$

where $\lambda_1 \sim \lambda_2 \sim 1$, $v \ll M$, the signature of the metric is chosen to be $(-, +, +, +)$. The corresponding vacuum solutions for the fields are given by the equations of motion and look like

$$ \phi_{vac} \approx M + \frac{\gamma v^8}{32\lambda_1 M^7}, \quad (2) $$

$$ h_{vac} \approx \frac{v}{\sqrt{2}} - \frac{\gamma v^5}{4\sqrt{2}\lambda_2 M^4}. \quad (3) $$

Let us suppose that $M = M_{GUT} \approx 10^{16} GeV$, $v = 250 GeV$ and $\gamma = 0.1$. The vacuum energy density

$$ \frac{\Lambda}{8\pi G} = \lambda_1 \left(\phi_{vac}^2 - M^2\right)^2 + \lambda_2 \left(h_{vac}^2 - \frac{v^2}{2}\right)^2 + \gamma\frac{h_{vac}^8}{\phi_{vac}^4} \quad (4) $$

where G is the gravitational constant, under these assumptions takes the value

$$\frac{\Lambda}{8\pi G} \approx \gamma \frac{v^8}{16 M_{GUT}^4} \sim 10^{-47} GeV^4, \tag{5}$$

which is exactly the value of the observed dark energy density. In some sense such a way of deriving the cosmological constant is similar to obtaining the value of the vacuum energy by combining the fundamental constants, – such examples are discussed in review [2]. One can also recall the "seesaw" mechanism for obtaining a small values of physical parameters using very different energy scales, which has been recently used in connection with the problem of cosmological constant [11, 12, 13]. At the same time, at least for the action (1), there can be other contributions to the vacuum energy density, for example, energy of the quantum fluctuations, which obviously neglects the value obtained above. Nevertheless, the idea discussed in this section can be used for constructing a model which gives the necessary value of the effective cosmological constant even in the case when the real vacuum energy density is much larger. We will discuss this model in the next section.

2 The model

Let us consider the action of the form

$$S = \int d^4x \sqrt{-g} \left[\alpha\phi R - \omega \frac{\partial^\mu \phi \partial_\mu \phi}{\phi} - \lambda_1 \left(\phi - M^2\right)^2 - \bar{\Lambda} - \frac{1}{48} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} - \right. \tag{6}$$
$$\left. - \left(D_\mu H\right)^\dagger D^\mu H - \lambda_2 \left(H^\dagger H - \frac{v^2}{2}\right)^2 + \gamma \left(H^\dagger H\right)^2 \left(\frac{H^\dagger H}{\phi}\right)^n + L_{SM-Higgs} \right],$$

where R is the four-dimensional curvature, ϕ is the Brans-Dicke field, ω is the dimensionless Brans-Dicke parameter, H is the Higgs field, $\bar{\Lambda} > 0$ is the "bare" energy density of the vacuum and is supposed to include, for example, contribution of quantum fluctuations, thus its value can be large (in this action $\bar{\Lambda} = \rho_{vac}$ – simply the energy density of the vacuum, in this sense it is not the cosmological constant defined by $\Lambda = 8\pi G \rho_{vac}$), $L_{SM-Higgs}$ is the Lagrangian of the Standard Model fields without Higgs's kinetic term and potential. The Lagrangian of the 3-form gauge field is also added to the action to make the cosmological constant in the Einstein equations be integration constant (see [14, 15]). Constants α, γ, λ_1 and λ_2 are dimensionless. We also suppose that $v \ll M$. Since we discuss a theory which includes gravity, we do not take into account possible issues concerning renormalizability of such theory. The potential containing Higgs field can be also represented in another form

$$\left[\lambda_2 - \gamma \left(\frac{H^\dagger H}{\phi}\right)^n\right] \left(H^\dagger H\right)^2 - \lambda_2 v^2 H^\dagger H + \frac{\lambda_2 v^4}{4}. \tag{7}$$

Of course, the constant term $\lambda_2 v^4/4$ can be incorporated into $\bar{\Lambda}$, but we retain it for simplicity.

Thus, we introduced explicit interaction between the Brans-Dicke and Higgs fields. It should be noted that the idea to consider interaction of the Higgs field with the fields related to gravity was already discussed in the literature, see some examples in papers [16, 17]. As for the use of the two interacting scalar fields in cosmology, such constructions are widely used in hybrid inflation models, see, for example, review [18]. The Brans-Dicke field itself was also discussed in cosmology [19].

We suppose that the vacuum expectation value of the field ϕ is

$$\phi_{vac} = M^2. \tag{8}$$

Correspondingly, from equation of motion for the Higgs field we get

$$\left(H^\dagger H\right)_{vac} \approx \frac{v^2}{2} + \frac{(n+2)\gamma v^{2n+2}}{2^{n+2}\lambda_2 M^{2n}}. \tag{9}$$

Equation for the field ϕ gives us

$$\alpha R = \gamma \left(H^\dagger H\right)^{n+2} \frac{n}{\phi_{vac}^{n+1}}, \tag{10}$$

which means that

$$R = \gamma \left(H^\dagger H\right)^{n+2} \frac{n}{\alpha\phi_{vac}^{n+1}}. \tag{11}$$

Thus, the value of the effective cosmological constant is

$$\Lambda_{eff} = \frac{\gamma n \left(H^\dagger H\right)_{vac}^{n+2}}{2\alpha\phi_{vac}^{n+1}}. \tag{12}$$

The solution of equations of motion for the 3-form gauge field is

$$F^{\mu\nu\rho\sigma} \sim c\epsilon^{\mu\nu\rho\sigma}, \tag{13}$$

where c is a constant, and contribution of this 3-form field to the action reduces to

$$-\frac{1}{48}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} \rightarrow +\frac{c^2}{2}. \tag{14}$$

The constant c in (13) is not fixed by the equations of motion for the 3-form field. It is fixed by the Einstein equations in accordance with (11). Indeed, it follows from the contracted Einstein equations that

$$\alpha\phi_{vac}R = 2\left(\tilde{\Lambda} - \frac{c^2}{2}\right), \tag{15}$$

where $\tilde{\Lambda}$ includes $\bar{\Lambda}$ and the constant contributions coming from Higgs and Brans-Dicke potentials. Finally

$$c^2 = 2\left(\tilde{\Lambda} - \alpha\phi_{vac}\Lambda_{eff}\right). \tag{16}$$

Thus, this field makes the whole set of equations of motion non-contradictory.

Now let us discuss possible energy scales of the model. The first one is associated with the scale at which Higgs acquires its vacuum expectation value. The second one, associated with M, can be chosen by the following reasons. In the beginning of inflation the gravity should be already "formed" at the classical level, otherwise we would be unable to perform classical analysis of the evolution. For the simplest model with the quartic inflaton potential the initial value of inflaton field is roughly equal to $10^{21}GeV$ [20]. We suppose that the field's ϕ vacuum expectation value is of the same order and take $M \sim 10^{21}GeV$. In this case its potential does not contribute considerably to the energy density of the Universe, which appears to be defined at that time only by the inflaton field. The vacuum expectation value of ϕ also defines the value of the Planck mass $M_{Pl} = \sqrt{\alpha\phi_{vac}}$.

Of course, there arises a question: why interaction of the Brans-Dicke field with the Higgs has the form used in (6)? Moreover, why these fields, one of which acquires its vacuum expectation value before the beginning of the inflation, interact one with another? We have no reasonable answers to these questions and note that action (6) should be interpreted only as a phenomenological model.

For $\alpha \sim 10^{-4}$, $n = 3/2$ and $\gamma = 1$ we get

$$\rho_{vac}^{eff} = \alpha\phi_{vac}\Lambda_{eff} \approx \frac{3\,(246^2/2)^{7/2}}{4(10^{63})}GeV^4 \approx 3.6 \cdot 10^{-48}GeV^4 \tag{17}$$

and

$$M_{Pl} = \sqrt{\alpha\phi_{vac}} \sim 10^{19}GeV. \tag{18}$$

We would like to note that the choice of the energy scales made above is not the only one possible. One can choose other values of parameters of the model. Moreover, the gravitational constant can be defined not by the Brans-Dicke field. Indeed, we can add the following term to the action (6):

$$S_{extra} = M_{Pl}^2 \int d^4x\sqrt{-g}R, \tag{19}$$

and suppose that the Brans-Dicke field is associated, for example, with the energy scale of the symmetry breaking in a possible theory of Grand Unification. In this case one can choose $M \sim M_{GUT} \sim 10^{16}GeV$, which clearly shows that for $\alpha \sim 1$ the gravitational constant is mainly defined by M_{Pl}, not by the Brans-Dicke field. This field is now simply the field non-minimally interacting with gravity. Nevertheless, if one chooses $\gamma \sim 1$, $\alpha \sim 1$, $n = 2.3$, the necessary value of Λ_{eff} (see (12)) also appears to be obtained.

In the end of this section we would like to say a few words about our choice of the vacuum expectation value for the field ϕ. The choice (8) is not the only possible. One can take, for example, a value slightly different from that used in (8). In this case the solution of corresponding equations of motion would lead to another value of an effective cosmological constant, as well as to another value of constant c. Thus, ϕ_{vac} is a free parameter in some sense. At the same time we suppose that Brans-Dicke field ϕ acquires its vacuum expectation value (and defines the Planck mass) at very high energies independently from the Higgs field (as well as from other fields), and it is defined only by the term $\lambda_1 (\phi - M^2)^2$ of the potential (or by an additional mechanism leading to (8)). This assumption seems to be reasonable from the physical point of view. When the Higgs field and the corresponding interaction with Brans-Dicke field come to play at much lower energies, the Higgs's vacuum expectation value appears to depend on ϕ_{vac}.

It should be also noted that the Higgs field was used only because its vacuum expectation value is quite convenient for obtaining the necessary value of the cosmological constant. One can choose another scalar field, even with a larger vacuum expectation value. Anyway, a Brans-Dicke field (or another scalar field non-minimally coupled to gravity) with a very large vacuum expectation value should be used, since it "connects" different energy scales.

3 Cosmological evolution

Now let us discuss how the Brans-Dicke field can affect cosmological evolution. First, we very briefly discuss the period of inflation for the simplest case of a single inflaton field. During the

slow-roll regime the energy density of the universe is supposed to be $V_{inf} \lesssim (10^{19} GeV)^4$. Let us consider contracted Einstein equation

$$M_{Pl}^2 R \approx 2V_{inf} + 2\lambda_1 \varphi^2, \tag{20}$$

and equation for the Brans-Dicke field

$$\alpha R \approx 2\lambda_1 \varphi. \tag{21}$$

Here $\phi = M^2 + \varphi$, $\varphi \ll M^2$, $\lambda_1 \sim 1$, we neglect the contribution of Λ_{eff} and time derivatives of the inflaton and Brans-Dicke field (indeed, during the slow-roll period the curvature R and inflaton field vary slowly, thus the Brans-Dicke field also varies slowly as follows from (21)). We also neglect contribution of the Higgs field, which appears to be reasonable for such energy scale.

Multiplying (21) by M^2 and combining with (20) we get

$$\varphi \approx \frac{V_{inf}}{\lambda_1 M^2} \lesssim 10^{-8} M^2. \tag{22}$$

We see that the assumption $\varphi \ll M^2$ is satisfied.

The contribution of the Brans-Dicke field to the energy density can be approximated by the relation

$$\lambda_1 \varphi^2 \sim \frac{V_{inf}}{M^4} V_{inf} \lesssim \frac{M_{Pl}^4}{M^4} V_{inf} \ll V_{inf}. \tag{23}$$

We see, that the Brans-Dicke field does not make a significant contribution to the energy density, and in this case inflation is driven by the inflaton. Moreover, the Brans-Dicke field self-tunes itself in an appropriate way to make the equation for the Brans-Dicke field be satisfied.

Of course, corresponding analysis should be made much more carefully, here we presented only rough reasonings.

Now let us turn to the evolution of the Universe at the present time. Indeed, the evolution is governed not only by the cosmological constant, but by ordinary and dark matter also. We denote the (average) energy-momentum tensor of ordinary and dark matter by $t_{\mu\nu}$.

Let us again consider contracted Einstein equation and equation for the Brans-Dicke field with $\phi = M^2 + \varphi$. We also suppose that $\varphi \ll M^2$ and neglect time derivatives of the Brans-Dicke field (indeed, Brans-Dicke's kinetic term takes the form $\sim \omega \frac{\partial^\mu \varphi \partial_\mu \varphi}{M^2}$, which can be dropped in comparison with $\lambda_1 \varphi^2$ for the evolution scale defined by Λ_{eff}). Below we will show that these assumptions indeed are satisfied (at the same time we should remember that actually field φ depends on time because of the time dependence of the trace of the energy-momentum tensor $t = t_\mu^\mu$).

The corresponding contracted Einstein equation and equation for the Brans-Dicke field look like

$$M_{Pl}^2 R \approx 2M_{Pl}^2 \Lambda_{eff} + 2\lambda_1 \varphi^2 - t. \tag{24}$$

$$M_{Pl}^2 R \approx 2M_{Pl}^2 \Lambda_{eff} + 2\lambda_1 M^2 \varphi, \tag{25}$$

from which it follows that

$$\varphi \approx -\frac{t}{2\lambda_1 M^2}. \tag{26}$$

We see that for the present average density of the Universe $\varphi \ll M^2$, and our assumption is indeed satisfied. The contribution of this field to the energy density

$$\lambda_1 \varphi^2 \sim \frac{t^2}{4\lambda_1 M^4},$$

which is much smaller even than $M_{Pl}^2 \Lambda_{eff}$ (we realize that $t \sim M_{Pl}^2 \Lambda_{eff}$), and thus can be neglected. Again we see that the Brans-Dicke field does not affect the evolution, its contribution to the energy-momentum tensor can be dropped, and again it self-tunes itself in accordance with the "ordinary" evolution governed by the Einstein equations.

In the end of this section it is necessary to discuss the problem concerning the value of constant c. Indeed, it is defined in accordance with equations (12) and (16), which were obtained for the case of absence of any matter except Higgs and Brans-Dicke fields in their vacuum states. At the same time c is a constant and does not depend on time. In the limit $x^0 \to \infty$ the ordinary and dark matter average densities tend to zero, $\varphi \to 0$, solution for the metric tends to dS_4 and only the cosmological constant contributes to the energy density and pressure (the value of $\bar{\Lambda}$ in (2) is supposed to be that after all the phase transitions such as electro-weak and QCD). Thus, the constant c should be such that equations of motion in this asymptotic case be satisfied, i.e. it is defined by the boundary conditions at the time infinity. This is exactly the value given by (12).

We would like to note that in this section only a very brief discussion of cosmological evolution is presented. One should make a thorough analysis to make sure that the existence of the Brans-Dicke field does not affect the cosmological evolution significantly. But we suppose that since the parameter M is very large, the Brans-Dicke field would self-tune itself corresponding to Einstein equations at all stages of the evolution, at least at the classical level. At the same time the mechanism discussed in Section 2 could "switch on" after the inflation period or even later (for example, if the scalar field coupled to gravity is not connected with definition of the Planck mass and acquires its vacuum expectation value after this stage) and could not produce any effect at these early stages. Anyway, at the present time, when the energy/mass densities of usual baryonic matter, dark matter and vacuum (defined by ρ_{eff}) are comparable, our mechanism indeed works.

4 Stability and Brans-Dicke–Higgs fields mixing

In this section we will discuss how this model can modify Higgs sector of the Standard model and its influence on the classical Newtonian gravity. To this end we consider second variation Lagrangian of the theory. Let us denote $g_{\mu\nu} = g_{\mu\nu}^0 + \frac{1}{M_{Pl}} h_{\mu\nu}$, where $g_{\mu\nu}^0$ is the background metric, $\phi = M^2 + \varphi$ and

$$H = \begin{pmatrix} 0 \\ \sqrt{(H^\dagger H)_{vac}} + \frac{\Phi}{\sqrt{2}} \end{pmatrix}, \tag{27}$$

then substitute it into action (6) and retain the terms quadratic in metric and scalar fields. We get

$$S = \int d^4 x \sqrt{-g^0} \left[L_2[h_{\mu\nu}] - \frac{\omega}{M^2} \partial_\mu \varphi \partial^\mu \varphi - \lambda_1 \varphi^2 - \frac{1}{2} \partial^\mu \Phi \partial_\mu \Phi - \frac{m_H^2}{2} \Phi^2 - \tag{28}$$

$$-\frac{1}{M_{Pl}} h^{\mu\nu} \alpha \left(\varphi R_{\mu\nu}^0 - \varphi \frac{1}{2} g_{\mu\nu}^0 R^0 - \nabla_\mu \nabla_\nu \varphi + g_{\mu\nu}^0 \nabla_\rho \nabla^\rho \varphi \right) - \gamma \frac{n(n+2)\sqrt{2} \left(H^\dagger H \right)_{vac}^{n+\frac{3}{2}}}{M^{2n+2}} \Phi \varphi +$$

$$+\frac{1}{2M_{Pl}} h^{\mu\nu} g_{\mu\nu}^0 \sqrt{2} \Phi \sqrt{(H^\dagger H)_{vac}} \left(\frac{\gamma(n+2) \left(H^\dagger H \right)_{vac}^{n+1}}{M^{2n}} - 2\lambda_2 \left(\left(H^\dagger H \right)_{vac} - \frac{v^2}{2} \right) \right) -$$

$$-\frac{1}{2M_{Pl}}h^{\mu\nu}g^0_{\mu\nu}\varphi\frac{n\gamma\left(H^\dagger H\right)^{n+2}_{vac}}{M^{2n+2}}+\frac{1}{2M_{Pl}}h^{\mu\nu}t_{\mu\nu}\Bigg].$$

Here $L_2[h_{\mu\nu}]$ is the Lagrangian containing terms of the second order in $h_{\mu\nu}$, ∇_μ is the covariant derivative with respect to the background metric $g^0_{\mu\nu}$, m_H is the mass of Higgs field Φ in the unitary gauge, $R^0_{\mu\nu}$ and R^0 contain only $g^0_{\mu\nu}$, $\omega \sim 1$, $\lambda_1 \sim 1$ and other parameters are the same as those used in Section 2. We neglect contributions $\sim \varphi^2$, $\sim \Phi^2$, coming from the term describing Higgs–Brans-Dicke fields interaction, in comparison with $\lambda_1\varphi^2$ and $m^2_H\Phi^2$ respectively. We also include interaction of the graviton with matter with the energy-momentum tensor $t_{\mu\nu}$.

With the help of equations (9), (11) for the background solution this action takes a simpler form, where we have also replaced $\left(H^\dagger H\right)_{vac}$ by $\frac{v^2}{2}$ in the term $\sim \Phi\varphi$

$$S = \int d^4x\sqrt{-g^0}\left[L_2[h_{\mu\nu}] - \frac{\omega}{M^2}\partial_\mu\varphi\partial^\mu\varphi - \lambda_1\varphi^2 - \frac{1}{2}\partial^\mu\Phi\partial_\mu\Phi - \frac{m^2_H}{2}\Phi^2 - \quad (29)$$
$$-\frac{1}{M_{Pl}}h^{\mu\nu}\alpha\left(\varphi R^0_{\mu\nu} - \nabla_\mu\nabla_\nu\varphi + g^0_{\mu\nu}\nabla_\rho\nabla^\rho\varphi\right) - \gamma\frac{n(n+2)v^{2n+3}}{2^{n+1}M^{2n+2}}\Phi\varphi + \frac{1}{2M_{Pl}}h^{\mu\nu}t_{\mu\nu}\right].$$

We see that some non-diagonal terms have vanished from the action. For simplicity, we neglect the effect of the cosmological constant. In this case $g^0_{\mu\nu} \to \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the flat Minkowski metric, $R^0_{\mu\nu} \to 0$ and $\nabla_\mu \to \partial_\mu$. In addition, let us make redefinition $h_{\mu\nu} \Rightarrow h_{\mu\nu} - \frac{\sqrt{\alpha}}{M}\eta_{\mu\nu}\varphi$. Substituting it into action (29), we get

$$S = \int d^4x\left[L_{FP}[h_{\mu\nu}] - \frac{1}{2}\left(\frac{2\omega+3\alpha}{M^2}\right)\partial_\mu\varphi\partial^\mu\varphi - \lambda_1\varphi^2 - \frac{1}{2}\partial^\mu\Phi\partial_\mu\Phi - \frac{m^2_H}{2}\Phi^2 - \quad (30)$$
$$-\gamma\frac{n(n+2)v^{2n+3}}{2^{n+1}M^{2n+2}}\Phi\varphi + \frac{1}{2M_{Pl}}h^{\mu\nu}t_{\mu\nu} - \frac{\sqrt{\alpha}}{2M_{Pl}M}\varphi\, t\right].$$

Here L_{FP} is the standard Fierz-Pauli Lagrangian. After redefining the fluctuations of the Brans-Dicke field as $\tilde\varphi = \frac{\sqrt{2\omega+3\alpha}}{M}\varphi$, (30) takes the form

$$S = \int d^4x\left[L_{FP}(h_{\mu\nu}) - \frac{1}{2}\partial_\mu\tilde\varphi\partial^\mu\tilde\varphi - \frac{m^2}{2}\tilde\varphi^2 - \frac{1}{2}\partial^\mu\Phi\partial_\mu\Phi - \frac{m^2_H}{2}\Phi^2 - \quad (31)$$
$$-\frac{b}{2}\Phi\tilde\varphi + \frac{1}{2M_{Pl}}h^{\mu\nu}t_{\mu\nu} - \frac{\sqrt{\alpha}}{\sqrt{2\omega+3\alpha}}\frac{1}{2M_{Pl}}\tilde\varphi\, t\right].$$

Where

$$m^2 = \frac{2\lambda_1 M^2}{2\omega+3\alpha} \sim M^2,$$

$$b = \frac{2\gamma n(n+2)v^{2n+3}}{2^{n+1}\sqrt{2\omega+3\alpha}M^{2n+1}} \sim \frac{v^6}{M^4}.$$

The Lagrangian for the scalar fields can be easily diagonalized with the help of the substitution

$$\Phi = \Phi'\cos\theta + \tilde\varphi'\sin\theta, \quad (32)$$
$$\tilde\varphi = \tilde\varphi'\cos\theta - \Phi'\sin\theta \quad (33)$$

with

$$\tan 2\theta = \frac{b}{m^2 - m_H^2}.$$

Since $\theta \ll 1$, we get

$$\Phi = \Phi' + \frac{b}{2m^2}\tilde{\varphi}', \tag{34}$$

$$\tilde{\varphi} = \tilde{\varphi}' - \frac{b}{2m^2}\Phi' \tag{35}$$

and

$$m_H'^2 = m_H^2 + m^2 O\left(\frac{b^2}{m^4}\right) \approx m_H^2, \tag{36}$$

$$m'^2 = m^2 + m^2 O\left(\frac{b^2}{m^4}\right) \approx m^2. \tag{37}$$

Here

$$\frac{b}{m^2} \sim \frac{v^6}{M^6} \sim 10^{-112}.$$

The action takes the form

$$S = \int d^4x \left[L_{FP}(h_{\mu\nu}) - \frac{1}{2}\partial_\mu\tilde{\varphi}'\partial^\mu\tilde{\varphi}' - \frac{m^2}{2}\tilde{\varphi}'^2 - \frac{1}{2}\partial^\mu\Phi'\partial_\mu\Phi' - \frac{m_H^2}{2}\Phi'^2 + \right. \tag{38}$$

$$\left. + \frac{1}{2M_{Pl}}h^{\mu\nu}t_{\mu\nu} - \frac{\sqrt{\alpha}}{\sqrt{2\omega+3\alpha}}\frac{1}{2M_{Pl}}\tilde{\varphi}'t + \frac{b\sqrt{\alpha}}{2m^2\sqrt{2\omega+3\alpha}}\frac{1}{2M_{Pl}}\Phi't \right].$$

Quadratic action (38) can be also obtained in another way. First, we can make a conformal rescaling in (6) and pass to the Einstein frame, then consider quadratic approximation for the Lagrangian of the scalar fields and fluctuations of the metric taking into account the equations for the background metric, then make the diagonalization with the help of (32), (33) and only finally pass to the flat metric.

We see that the linearized theory does not contain tachyons or ghosts, i.e. it is stable. The Brans-Dicke field and Higgs field appear to be mixed, which leads to new interactions, for example, of the Higgs field with matter through the trace of the energy-momentum tensor t. But the corresponding new interactions can be completely neglected because of the suppression by the factor 10^{-112} for our choice of the parameters of the model. We also see that the mass of the Brans-Dicke field $m \sim 10^{21} GeV$, so that this field decouples from the low-energy effective theory. Thus, in the linear approximation, in fact, we have ordinary tensor massless gravity.

5   Conclusion

In this paper we discussed a model which provides the necessary value of the effective cosmological constant at the classical level. We used interacting Higgs and Brans-Dicke fields and the 3-form gauge field. It is necessary to note that the Higgs and Brans-Dicke fields are not the only possible fields which can be used in the mechanism described above. The Higgs field was used because of the value of the Standard Model's symmetry breaking scale, while Brans-Dicke field was used because the corresponding theory is one of the most known which can be

used to define the Planck mass within the framework of the classical field theory. Obviously one can use any two interacting (in an appropriate way) scalar fields with different vacuum expectation values, one of which is non-minimally coupled to gravity, or even only one scalar field, non-minimally coupled to gravity. But in the latter case one should either introduce some energy scale "by hands" (in our case it is provided by the Higgs mechanism), or use potentials including, for example, exponential terms to make the hierarchy of scales. Indeed, the idea is simple – the effective cosmological constant appears due to the interaction with Brans-Dicke field, and the Higgs mechanism sets only the second energy scale. But since we already have this scale, it is reasonable to use it, than to introduce a new one. At the same time the 3-form gauge field is necessary to make the "bare" cosmological constant be an integration constant.

We think that it is also interesting to make a thorough study of how the Brans-Dicke field in our model affects the whole cosmological evolution. But this problem calls for further investigations.

Acknowledgments

References

[1] S. Weinberg, Rev. Mod. Phys. **61** (1989) 1.

[2] V. Sahni and A.A. Starobinsky, Int. J. Mod. Phys. D **9** (2000) 373 [arXiv:astro-ph/9904398].

[3] V. Sahni, Lect. Notes Phys. **653** (2004) 141 [arXiv:astro-ph/0403324].

[4] A.D. Dolgov, arXiv:hep-ph/0405089.

[5] A.D. Dolgov, Int. J. Mod. Phys. A **20** (2005) 2403.

[6] T. Padmanabhan, AIP Conf. Proc. **861** (2006) 179 [arXiv:astro-ph/0603114].

[7] A.D. Dolgov, arXiv:hep-ph/0606230.

[8] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **15** (2006) 2105 [arXiv:astro-ph/0610026].

[9] A.D. Dolgov, AIP Conf. Proc. **910** (2007) 3.

[10] E.J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15** (2006) 1753 [arXiv:hep-th/0603057].

[11] M. McGuigan, arXiv:hep-th/0602112.

[12] V.V. Kiselev and S.A. Timofeev, arXiv:0710.1685 [hep-th].

[13] V.V. Kiselev and S.A. Timofeev, arXiv:0710.2204 [hep-th].

[14] A. Aurilia, H. Nicolai and P.K. Townsend, TH.2884–CERN.

[15] M. Henneaux and C. Teitelboim, Phys. Lett. B **143** (1984) 415.

[16] J.J. van der Bij, Acta Phys. Polon. B **25** (1994) 827.

[17] O. Bertolami and R. Rosenfeld, arXiv:0708.1784 [hep-ph].

[18] A. Linde, arXiv:0705.0164 [hep-th].

[19] J. Garcia-Bellido, A.D. Linde and D.A. Linde, Phys. Rev. D **50** (1994) 730 [arXiv:astro-ph/9312039].

[20] V. Rubakov, PoS **RTN2005** (2005) 003.